CHAPTER

1 Relations and Functions

Relations

If A and B are two non-empty sets, then a relation R from A to B is a subset of $A \times B$.

Representation of a Relation

Roster form: In this form, we represent the relation by the set of all ordered pairs belongs to R.

Set-builder form: In this form, we represent the relation R from set A to set B as

$R = \{(a, b) : a \in A, b \in B$ and the rule which relate the elements of A and $B\}$.

Domain, Codomain and Range of a Relation

Let R be a relation from a non-empty set A to a non-empty set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs belonging to R is called the range of R. Also, the set B is called the codomain of relation R.

Thus, domain of $R = \{a : (a, b) \in R\}$ and range of $R = \{b : (a, b) \in R\}$

Types of Relations

Empty or Void Relation: As $\phi \subset A \times A$, for any set A, so ϕ is a relation on A, called the empty or void relation.

Universal Relation: Since, $A \times A \subseteq A \times A$, so $A \times A$ is a relation on A, called the universal relation.

Identity Relation: The relation $I_A = \{(a, a): a \in A\}$ is called the identity relation on A.

Reflexive Relation: A relation R on a set A is said to be reflexive relation, if every element of A is related to itself.

Thus, $(a, a) \in R, \forall a \in A \Rightarrow R$ is reflexive.

Symmetric Relation: A relation R on a set A is said to be symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R, \forall a, b \in A$

i.e. $a R b \Rightarrow bRa, \forall a, b \in \mathrm{A}$

Transitive Relation: A relation R on a set A is said to be transitive relation, iff $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad (a, c) \in R, \forall a, b, c \in A$

Equivalence Relation

A relation R on a set A is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on A.

Functions

Let A and B be two non-empty sets, then a function f from set A to set B is a rule which associates each element of A to a unique element of B.

Domain, Codomain and Range of a Function

If $f : A \to B$ is a function from A to B, then

(*i*) the set A is called the domain of $f(x)$.

(*ii*) the set B is called the codomain of $f(x)$.

(*iii*) the subset of B containing only the images of elements of A is called the range of $f(x)$.

Number of Functions

Let X and Y be two finite sets having m and n elements respectively. Then each element of set X can be associated to any one of n elements of set Y. So, total number of functions from set X to set Y is n^m.

Number of One-One Functions

Let A and B are finite sets having m and n elements repectively, then the number of one-one functions from A to B is $\begin{cases} {}^nP_m, n \geq m \\ 0, n < m \end{cases}$

$$= \begin{cases} n(n-1)(n-2)...(n-(m-1)), n \geq m \\ 0, \quad n < m \end{cases}$$

Number of Onto (or Surjective) Functions

Let A and B are finite sets having m and n elements respectively, then number of onto (or surjective) functions from A to B is

$$= \begin{cases} n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + ..., n < m \\ n!, \quad n = m \\ 0, \quad n > m \end{cases}$$

Number of Bijective Functions

Let A and B are finite sets having m and n elements respectively, them number of bijective functions from A to B is

$$= \begin{cases} n!, \text{ if } n = m \\ 0, \text{ if } n > m \text{ or } n < m \end{cases}$$

Properties of Greatest Integer Function

(*i*) $[x + n] = n + [x], n \in I$

(*ii*) $[-x] = -[x], x \in I$

(*iii*) $[-x] = -[x] - 1, x \notin I$

(*iv*) $[x] \geq n \Rightarrow x \geq n, n \in I$

(*v*) $[x] > n \Rightarrow x \geq n + 1, n \in I$

(*vi*) $[x] \leq n \Rightarrow x < n + 1, n \in I$

(*vii*) $[x] < n \Rightarrow x < n, n \in I$

(*viii*) $[x + y] = [x] + [y + x - [x]]$ for all $x, y \in R$

(*ix*) $[x + y] \geq [x] + [y]$

Important Points To Be Remembered

(*i*) Constant function is periodic with no fundamental period.

(*ii*) If $f(x)$ is periodic with period T, then $\dfrac{1}{f(x)}$ and $\sqrt{f(x)}$ are also periodic with same period T.

(*iii*) If $f(x)$ is periodic with period T, then $kf(ax + b)$ is periodic with period $\dfrac{T}{|a|}$, where $a, b, k \in R$ and $a, k \neq 0$.

Properties of Even and Odd Functions

(*i*) *gof* or *fog* is even, if both f and g are even or if f is odd and g is even or if f is even and g is odd.

(*ii*) *gof* or *fog* is odd, if both of f and g are odd.

(*iii*) If $f(x)$ is an even function, then $\dfrac{d}{dx}f(x)$ is an odd function and if $f(x)$ is an odd function, then $\dfrac{d}{dx}f(x)$ is an even function.

(*iv*) The graph of an even function is symmetrical about Y-axis.

(*v*) The graph of an odd function is symmetrical about origin or symmetrical in opposite quadrants.

(*vi*) An even function can never be one-one, however an odd function may or may not be one-one.

Properties of Inverse Function

(*a*) The inverse of a bijection is unique.

(*b*) If $f : A \rightarrow B$ is a bijection and $g : B \rightarrow A$ is the inverse of f, then $fog = I_B$ and $gof = I_A$, where I_A & I_B are identity functions on the sets A & B respectively. If $fof = I$, then f is inverse of itself.

(*c*) The inverse of a bijection is also a bijection.

(*d*) If f & g are two bijections $f : A \rightarrow B$, $g : B \rightarrow C$ & *gof* exist, then the inverse of *gof* also exists and $(gof)^{-1} = f^{-1} og^{-1}$.

(*e*) The graph of f^{-1} obtained by reflecting the graph of f about the line $y = x$.

General

If x, y are independent variables, then :

(*a*) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k\ell nx$

(*b*) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n, n \in R$ or $f(x) = 0$

(*c*) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$ or $f(x) = 0$

(*d*) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$, where k is a constant.